Intersection Theory

Sheet 5

will be discussed on June 20

Let X be a variety.

Exercise 1. Let E be a vector bundle on X. Show that

$$c_i(E^\vee) = (-1)^i c_i(E) \ .$$

Hint: Use the splitting principle.

Exercise 2. Let E be a vector bundle on X of rank r and let L be a line bundle on X. Show that

$$c_r(E \otimes L) = \sum_{i=0}^{r} c_i(E) c_1(L)^{r-i} \ .$$

Hint: Use the splitting principle.

Exercise 3. Let E and F be vector bundles on X of ranks r and s, respectively. Show that

$$c_1(E \otimes F) = s \cdot c_1(E) + r \cdot c_1(F) \ .$$

Hint: Use the splitting principle.

Exercise 4. Let E be a vector bundle on X of rank r with a nowhere vanishing section $s\colon X \to E$. Show that $c_r(E) = 0$.

Hint: Use the splitting principle.

For $\Phi \in \mathrm{Hom}(\mathrm{CH}_{\dim X}(X), \mathrm{CH}_0(X))$, let us write $\int_X \Phi := \deg(\Phi \cap [X])$.

Exercise 5. (a) Compute the Chern classes of $T_{\mathbb{P}^n}$, the tangent bundle of $\mathbb{P}^n$.

(b) Let $S \subset \mathbb{P}^3$ be a smooth surface of degree d. Compute

$$\int_S c_2(T_S) \quad \text{and} \quad \int_S c_1(T_S)^2 \ .$$