Intersection Theory

Sheet 4

will be discussed on May 30

Exercise 1. Let D be an effective Cartier divisor on a variety X. Show that the following three subsets of X are the same:

- the support of D
- the support of the Weil divisor associated to D
- the underlying set of the closed subscheme determined by D

Exercise 2. Let $X = \{x_0^2 = x_1 x_2\} \subset \mathbb{P}^3$. For $i \in \{0,1\}$, we consider the Cartier divisors $D_i = \{x_i = 0\} \subset X$.

(a) Determine the excess of intersection $\varepsilon(D_0, D_1)$.

(b) Compute the blow-up $\pi \colon \tilde{X} \to X$ in $D_0 \cap D_1$.

(c) Describe how the proof of $D_0 \cdot D_1 = D_1 \cdot D_0$ proceeds in this example.

Exercise 3. Let D_1 and D_2 be pseudo-divisors on a variety X, and let $\alpha \in \mathcal{Z}_k(X)$. Show that

$$D_1 \cdot (D_2 \cdot \alpha) = D_2 \cdot (D_1 \cdot \alpha)$$

in $\mathrm{CH}_{k-2}(|D_1| \cap |D_2| \cap |\alpha|)$.

Exercise 4. Let $f \colon X \to C$ be a dominant morphism from a variety X to a smooth curve C. Let $p \in C$ be a closed point.

(a) Show that $D = f^{-1}(p)$ is an effective Cartier divisor on X.

(b) Show that $D \cdot (D \cdot \alpha) = 0$ in $\mathrm{CH}_{k-2}(|D| \cap |\alpha|)$ for all $\alpha \in \mathcal{Z}_k(X)$.

Exercise 5. Let $\pi \colon L \to X$ be a line bundle.

(a) Show that the zero section $s \colon X \to L$ is the inclusion of an effective Cartier divisor.

(b) Prove that $s^* \circ \pi^*$ is the identity on $\mathrm{CH}_k(X)$.

(c) Conclude that $\pi^* \colon \mathrm{CH}_k(X) \to \mathrm{CH}_{k+1}(L)$ is an isomorphism.