Intersection Theory

Sheet 1

will be discussed on April 19

Exercise 1. Let $X = \{x^2 = y^5\} \subset \mathbb{A}^2_{\mathbb{C}}$. Compute the order of vanishing of $\frac{x}{y^2}$ at $(0, 0)$.

Exercise 2. Let X be a variety and $D \subset X$ a prime divisor. We consider the local ring $A = \mathcal{O}_{X,D}$ with maximal ideal $\mathfrak{m}$.

(a) Show that

$$\text{length}_A(A/f) \geq \max\{i \mid f \in \mathfrak{m}^i\}$$

for all $f \in A$.

(b) Show that if

$$\text{length}_A(A/f) = \max\{i \mid f \in \mathfrak{m}^i\}$$

for all $f \in A$, then A is a discrete valuation ring.

Exercise 3. Compute the Chow groups of $X = \{x_0x_1x_2x_3 = 0\} \subset \mathbb{P}^3_{\mathbb{C}}$.

Exercise 4. Let X be a variety over $\mathbb{C}$. Show that $\text{CH}_0(X \times \mathbb{A}^1) = 0$.

Exercise 5. Let $X \subset \mathbb{P}^n_{\mathbb{C}}$ be a projective variety. Let $f, g \in \mathbb{C}[x_0, \ldots, x_n]$ be homogeneous polynomials of the same degree d which do not completely vanish on X, so we have $\frac{f}{g} \in \mathbb{C}(X)^*$. Let D_+ and D_- be the divisors associated to the subschemes $X \cap \{f = 0\}$ and $X \cap \{g = 0\}$. Show that

$$\text{div}\left(\frac{f}{g}\right) = D_+ - D_- \ .$$